Tutorial 6

Find the continuity points of the following functions. Questions 1,2 are from our textbook[Bartle] at p.129. Questions 3,4 are from [Bartle] p.127,128.

1. (a)
$$h : \mathbb{R} \to \mathbb{R}, \quad h(x) := \lfloor x \rfloor$$
where $\lfloor x \rfloor := \max\{n \in \mathbb{Z} : n \le x\}$, e.g. $\lfloor -1.5 \rfloor = -2$

   (b)
$$h : \mathbb{R} \to \mathbb{R}, \quad h(x) := x\lfloor x \rfloor$$

   (c)
$$h : \mathbb{R} \setminus \{0\} \to \mathbb{R}, \quad h(x) := \lfloor \frac{1}{x} \rfloor$$

2.
$$g : \mathbb{R} \to \mathbb{R}, \quad g(x) := \begin{cases} 2x & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \\ x+3 & \text{otherwise} \end{cases}$$

3.
$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) := \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$$

4.
$$f : (0, \infty) \to \mathbb{R}, \quad f(x) := \begin{cases} 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \\ \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ with } \gcd(m,n) = 1 \end{cases}$$

5. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = f(2x)$ for every $x \in \mathbb{R}$. Show that f is a constant function.

6. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function satisfying $f(x + y) = f(x) + f(y)$ for every $x, y \in \mathbb{R}$ and $\lim_{x \to 0} f(x) = l \in \mathbb{R}$. Show that $l = 0$, f is continuous and $f(x) = cx$ for some constant c.